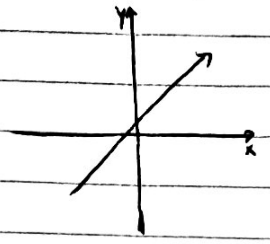
MAT1322
Lecture 16

Surfaces in 3D

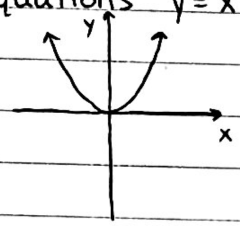
13/03/2017

In 2D, we used curves to describe functions.

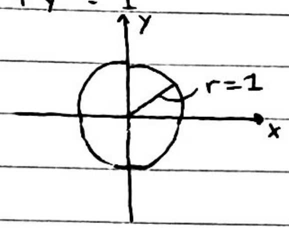
Eg: 1) linear equations $y = 3x + 1$ (tangent lines)



2) quadratic equations $y = x^2$



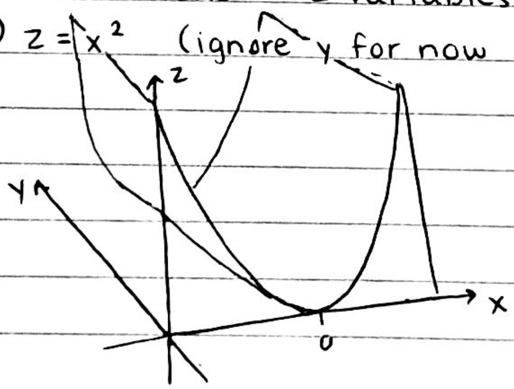
3) circles $x^2 + y^2 = 1$ ← radius^2



Now, in 3D:

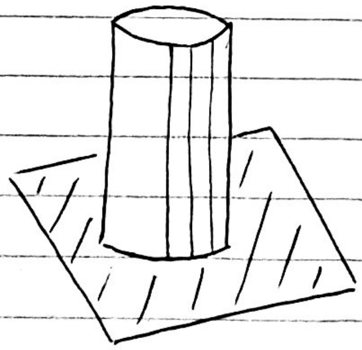
We have functions in 2 variables, $f(x, y) = z$ ← 3rd variable

1) $z = x^2$ (ignore y for now



* every point on the xy plane has a height *

2) $x^2 + y^2 = 1^2$ (circle on xy plane)

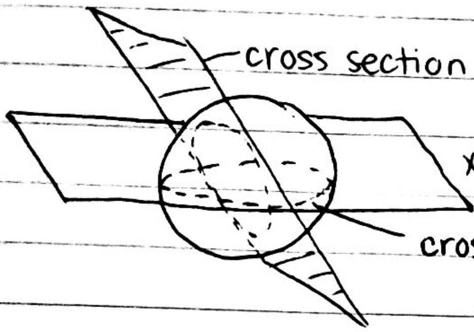


a cylinder!

- the parallel lines are called <u>rulings</u> or <u>traces</u>

Now, using x, y, and z:

(1) $x^2+y^2+z^2=1$ * a sphere



(2) $x^2+\frac{y^2}{9}+\frac{z^2}{4}=1$ → Ellipsoid

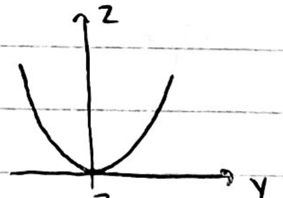
- google pic

* Traces are ellipses.

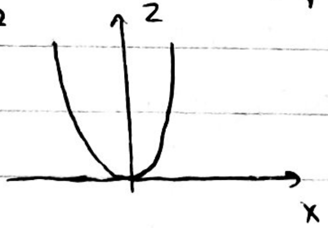
Try to sketch a curve: $z=4x^2+y^2$

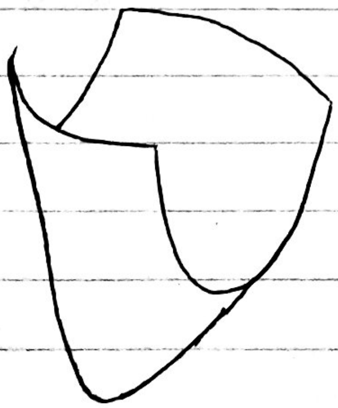
- fix one variable (slice the surface up at a vertical plane)

a) fix $x=0$: $z=y^2$



fix $y=0$: $z=4x^2$





PARABALOID

- traces are parabolas

SURFACES TO REMEMBER (compare to pg 837)

name	typical equation	traces
ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ a, b, c describe the dimensions	ellipses
elliptic paraballoid	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$	horizontal: ellipses vertical: parabolas
hyperbolic paraballoid	$\frac{z^2}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$	horizontal: hyperboles vertical: parabolas
cone	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ $\frac{z}{c} = \pm\sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2}}$ gives both parts	horizontal $(z=0)$: ellipses vertical: parabolas $(x = 0, 1, 2, \ldots$ $y = 0, 1, 2, \ldots)$